MEMORANDUM REPORT ARBRL-MR-03348
(Supersedes IMR No. 660)

MATHEMATICS OF MULTIDART PROBABILITY PREDICTIONS

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April 1984

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER MEMORANDUM REPORT ARBRL-MR-03348	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MATHEMATICS OF MULTIDART PROBABILITY PREDICTIONS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lawrence D. Johnson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory, ARDC ATTN: DRSMC-BLB(A) Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L162603AH18
11. CONTROLLING OFFICE NAME AND ADDRESS US Army AMCCOM, ARDC Ballistic Research Laboratory, ATTN: DRSMC-BLA-S(A) Aberdeen Proving Ground, MD 21005		12. REPORT DATE April 1984
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES This report supersedes BRL IMR-660, dated August 1979.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Probability Hit Multidart Patterns		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A general equation has been derived which can be used to predict the hit and kill probability of multidart single shot patterns, as well as single dart multishot patterns.		

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INTRODUCTION

This note discusses the mathematics associated with predicting the hit and kill probabilities associated with multidart rounds. For the sake of brevity, we will dispense with the traditional discussion of definitions pertaining to fixed bias, variable bias, and round-to-round errors. Instead, we will proceed directly to the problem at hand.

DERIVATION

Patterns

The patterns that are assumed are basically circular. In the "hollow" pattern the darts have individual variations, but their mean points of impact are assumed to fall on the circumference of a circle of radius r, equidistant to each other. A "dense" pattern is a hollow pattern with an additional dart in the center of the circle. Figure 1 schematically depicts the patterns considered.

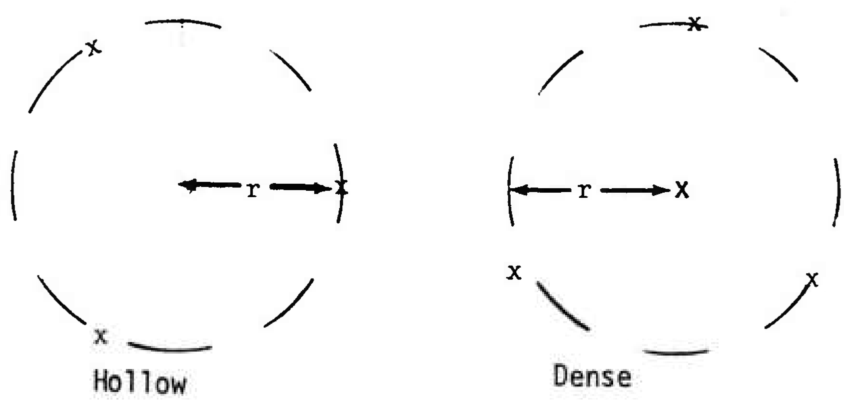

Figure 1. Patterns

Since the individual darts may vary about their mean point, they are not necessarily equidistant either from the center or from each other, for a particular event.

Probability Derivation

Let η_x,η_y represent the coordinates of the center of the circle. If the position of the center has a distribution similar to a single shot system,* its probability density function takes the form:

$$f(\eta_x,\eta_y) = \frac{1}{2\pi\sigma_{\eta_x}\sigma_{\eta_y}} \cdot \exp\left\{-\frac{1}{2}\left[\frac{(\eta_x - \mu_{\eta_x})^2}{\sigma^2_{\eta_x}} + \frac{(\eta_y - \mu_{\eta_y})^2}{\sigma^2_{\eta_y}}\right]\right\} \tag{1}$$

where $\mu_{\eta_x},\mu_{\eta_y} \triangleq$ fixed bias of system

$\sigma^2_{\eta_x},\sigma^2_{\eta_y} \triangleq$ variance of the system.

Assuming that the individual darts have a normal distribution about their mean points, their probability density functions are given by

$$f_i(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \cdot \exp\left\{-\frac{1}{2}\left[\frac{(x - g_{x_i}(r,\theta,\eta_x))^2}{\sigma^2_x} + \frac{(y - g_{y_i}(r,\theta,\eta_y))^2}{\sigma^2_y}\right]\right\} \tag{2}$$

where x,y = coordinates of the impact point

$g_{x_i}(r,\theta,\eta_x) = r\cos\left[\theta + (\frac{i-1}{N})2\pi\right] + \eta_x$; i.e., the mean point of impact for the ith dart in the x direction

$g_{y_i}(r,\theta,\eta_y) = r\sin\left[\theta + (\frac{i-1}{N})2\pi\right] + \eta_y$; the mean point of impact for the ith dart in the y direction

N = number of darts on the periphery of the circle of radius r.

i = the particular dart being examined

θ = angular orientation of the pattern

This is a desirable assumption since it allows for the degenerate case, i.e., a dense pattern having only one dart.

It should be noticed that for a dense pattern, the center dart has the density function defined by Equation (1). Also the density function for the pattern's orientation is assumed to be uniformly distributed,* i.e.,

$$\tilde{f}(\theta) = \frac{1}{2\pi} \; . \tag{3}$$

The probability of missing a target with all darts in the pattern for a given impact center η_x, η_y and orientation θ is

$$P_M\Big|_{\eta_x,\eta_y,\theta} = \left[1 - \overline{P}(\eta_x,\eta_y)\right]^j * \prod_{i=1}^{N} \left[1 - \iint_A f_i(x,y)\, dx\, dy\right] \tag{4}$$

where $\overline{P}(\eta_x,\eta_y)$ = 1.0 if point η_x,η_y is on target area and equals zero if not

j = 0 for hollow pattern and 1 for dense pattern

A = presented area of target.

Since all possible center of circle coordinates and orientations must be accounted for,

$$P_M = \int_0^{2\pi} \int_{-\infty}^{\infty} \int \left\{ \left[1 - \overline{P}(\eta_x,\eta_y)\right]^j \prod_{i=1}^{N} \left[1 - \iint_A f_i(x,y)\, dx\, dy\right] \right\} f(\eta_x,\eta_y) \tilde{f}(\theta)\, d\eta_x\, d\eta_y\, d\theta \tag{5}$$

and finally, the probability of hitting the target at least once is

$$P_H = 1 - P_M \; . \tag{6}$$

The probability of killing the target follows the same lines except that the density functions $f_i(x,y)$ must be multiplied by the probability of killing the target given an impact point x,y and $\overline{P}(\eta_x,\eta_y)$ is multiplied by a similar conditional kill value.

It is currently envisioned that the orientation of the round, when chambered, will be arbitrary; thus, the orientation of the pattern is assumed to be uniformly distributed.

It is of interest to note that Equations (5) and (6) are merely generalizations of the probability of hitting a target firing N rounds with a single shot system. In this case, $i = r = 0$ and η_x, η_y are the variable biases of the system. Under these conditions Equation (5) reduces to

$$P_M = \int_{-\infty}^{\infty}\int \left[1 - \int_A\int f(x,y)\, dx\, dy\right]^N f(\eta_x,\eta_y)\, d\eta_x\, d\eta_y \qquad (7)$$

SUMMARY

A general equation has been derived which predicts the hit and kill probability of multidart single shot patterns and single dart multishot patterns.

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